



MECHANICS OF SOLIDS (ME F211)







Mechanics of Solids

Chapter-1

Fundamental principles of mechanics







G Force

- Moment
- Types of loading
- □ Types of Support & related Reactions
- □ Free Body Diagrams
- Equilibrium Conditions
- Trusses & Frames
- Friction



Analysis of Engineering systems

Study of forces

(Tensile, Compressive, Shear, Torque, Moments)

Study of motion

(Straight, curvilinear, displacement, velocity, acceleration.)

Study of deformation

(Elongation, compression, twisting)

Application of laws relating the forces to the deformation

In some special cases one or more above mentioned steps may become trivial

e.g. For rigid bodies deformation will be negligible.

If system is at rest, position of system will be independent of time.



Step-By-Step procedure to solve problems in mechanics of solids

- □ Select actual/real system of interest.
- □ Make assumptions regarding desired characteristics of the system
- Develop idealized model of the system (Structural and Machine elements)
- □ Apply principles of mechanics to the idealized model to compare these calculated results with the behaviour of the actual system
- □ If the results (calculated and actual) differ widely repeat above steps till satisfactory idealized model is obtained.



Definitions

- □ Body: A collection of particles is called a 'body'. It may be a rigid body or an elastic or deformable body.
- □ **Rigid Body:** The particles in a rigid body are so firmly connected together that their relative positions do not change irrespective of the forces acting on it. Thus the size and shape of a rigid body are always maintained constant
- □ Elastic Body: A body whose size and shape can change under forces is a *deformable* body. When the size and shape can be regained on removal of forces, the body is called an *elastic body*.



Definitions

Scalar Quantity: A quantity which is fully described by its magnitude only is a scalar. Arithmetical operations apply to scalars. Examples are: Time, mass, area and speed.

Vector Quantity: A quantity which is described by its magnitude and also its direction is a vector. Operations of vector algebra are applicable to vectors. Examples are: Force, velocity, moment of a force and displacement.

Force: In physics, a net force acting on a body causes that body to accelerate; that is, to change its velocity. The concept appeared first in the second law of motion of classical mechanics.



Force

There are three basic kinds of forces as mentioned below

Tensile force or pull

Compressive force or push

□ Shear force



Tensile Force or Pull

When equal and opposite forces are applied at the ends of a rod or a bar away from the ends, along its axis, they tend to pull the rod or bar. This kind of a force is called a tensile force or tension.





Compressive Force or Push

When equal and opposite forces act at the ends of a rod or a bar towards the ends along its axis, they tend to push the rod or the bar. This kind of force is called a compressive force or compression.





Shear Force

When equal and opposite forces act on the parallel faces of a body, shear occurs on these planes. This tends to cause an angular deformation as shown.









System of Forces

Coplanar

Collinear, Concurrent, Parallel, Non-concurrent, Non-concurrent & Non-Parallel

Non-Coplanar

Concurrent, Parallel, Non-concurrent & Non-Parallel





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Resultant Force

If a force system acting on a body can be replaced by a single force, with exactly the same effect on the body, this single force is said to be the 'resultant' of the force system.



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Superposition Principal

If a system is acted up on a set of forces, then the net effect of these forces is equal to the summation net effect of individual force.

This principle is applicable for linear systems.





The moment of force

The moment of force **F** about a point O is r X **F**.





The moment of force

- □ The moment itself is a vector quantity.
- □ Its direction is perpendicular to the plane determined by *OP* and *F*.
- □ The sense is fixed by the right hand rule
- □ From calculus the magnitude of the cross product $r \times F = F r \sin \phi$, Where $r \sin \phi$ is the perpendicular distance between point *O* and vector *F*.







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Moment, *M* = *F* X *X*





Example 1: Moment





Solution of Example 1

 $M_o = 15 \sin 30 \text{ kN X 4 m} = 30 \text{ kN-m}$ (CCW)



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Example 3: Moment

 $M_o = 10 \text{ kN X 2 m} = 20 \text{ kN-m}$ (CW)





CALCULATE THE MOMENT 400 N FORCE ABOUT THE POINT *O*





Couple

A special case of moments is a couple. A couple consists of two parallel forces that are equal in magnitude, opposite in direction. It does not produce any translation, only rotation. The resultant force of a couple is zero. BUT, the resultant of a couple is not zero; it is a pure moment.





About couple

- □ This result is independent of the location of B.
- Moment of a couple is the same about all points in space.
- A couple may be characterized by a moment vector without specification of the moment center B, with magnitude Fd.
- □ Encircling arrow indicates moment of a couple.





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Types of Loading

1. Concentrated Load / Point Load





Types of Loading

2. Uniformly Distributed Load (UDL)





Types of Loading

3. Uniformly Varying Load (UVL)





Types of Loading

4. Moment (Pure Bending Moment)





Types of Loading

5. Moment (Twisting Moment)





Support & Reaction at the Support





Support & Reaction at the Support

2. Simple Support with hinge or pin at A & B






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Support & Reaction at the Support

5. Smooth/Frictionless Support



Support & Reaction at the Support

6. Friction Support





Equilibrium Conditions

If the resultant force acting on a particle is zero then that can be called as equilibrium

Dynamic Equilibrium

The body is said to be in equilibrium condition when the acceleration is zero

Static Equilibrium

The static body is in equilibrium condition if the resultant force acting on it is zero



Necessary and sufficient condition for body to be in Equilibrium

Summation of all the FORCES should be zero

Σ F = 0

Summation of all the MOMENTS of all the forces about any arbitrary point should be zero

Σ M = 0



Two Force Member

Two forces can't have random orientation, must be along $AB \& F_A = F_B$



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Three Force Member

- Three forces can't have random orientation
- All must lie in the plane ABC if total moment about each of the points A, B & C is to vanish.
- They must all intersect in a common point *O*.





Statically Determinate System

If it is possible to determine all the forces involved by using only the equilibrium requirements without regard to deformations, such systems are called <u>Statically Determinate</u>.

Use of equilibrium conditions to solve for unknown forces from known forces.



Statically Indeterminate System



Statically Indeterminate Situation

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Free Body Diagram (FBD)

The sketch of the isolated component from a system / and all the external forces acting on it is often called a freebody diagram



Free Body Diagram (FBD)









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Example

Determine the forces at B and C

Pin Pin 45 Bolts 20 kN 3 m

Idealization

- 1. Neglect the friction at pin joints
- 2. Ignore the self weight of rods



Free Body Diagram of the Frame





Free body diagrams of bar *BD* and *CD* i.e. Isolation from the system





Bolted joint is considered as pin joint



- $\Sigma F_y = 0$
- $F_{B} \sin 45 20 = 0$
- F_B = 28.28 kN
- $\Sigma F_x = 0$
- $F_{\rm C} F_{\rm B} \cos 45 = 0$

 $F_c = 20 \text{ kN}$



Trusses

Trusses are designed to support the loads and are usually stationary, fully constrained structures.

All the members of a truss are <u>**TWO FORCE</u>** members</u>

Applications of Truss

- Construction of roof
- Bridges
- Transmission line towers.







Construction of roof



Transmission line towers



Bridges

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Relation between number of joints (*j*) and member (*m*) in trusses

- m=2j-3
 - If a truss obeys the equation, is called **Perfect truss** (just rigid truss)
 - □ If no. of members are more than this equation then it is called **Redundant truss** (over rigid truss)
 - If no. of members are less than this equation then it is called
 Deficient truss (under rigid truss)



Methods of Analysis of Truss

- 1. Joint equilibrium method
- 2. Section equilibrium method



Joint equilibrium method

- In this method using FBD for each joint, equilibrium of the joints of the truss is consider one at a time.
- At each joint it is necessary to satisfy the static equilibrium equations.
- This method is suitable, when it is asked to calculate forces in all the members of the truss
- □ Only two static equilibrium equations will be satisfied at a time because $\Sigma M = 0$ will always be satisfied for concurrent force system.
- Only two unknown forces will be determined at a time.



Step by step procedure of joint equilibrium method

- 1. Draw a FBD of an entire truss showing the external forces acting on it. (i.e. applied forces/loads and reactions)
- 2. Obtain the magnitude and direction of reactions by using the static equilibrium equations.
- Select a joint where only two unknown forces exist and draw its FBD assuming all unknown forces tensile.
- 4. Apply the equilibrium equations to the joint, which will give the forces in the members.



Step by step procedure of joint equilibrium method

- 5. If the obtained value of a force is "- ve" then our assumed direction is wrong. Change the nature of force.
- 6. Apply the above steps to each joint and determine the forces.
- 7. At the end, redraw the truss and show the correct direction as well as its magnitude of force in each member.



Find the forces in each member of given truss

W = 10kN and F = 2kN. Length of each member of the truss is 1m.









FBD of Joint 'B'



Equilibrium of Joint 'B'

 $\Sigma F_X = 0 = F + F_{BC} \times \cos(60) - F_{AB} \times \cos(60)$ $\Sigma F_Y = 0 = -W - F_{BC} \times \sin(60) - F_{AB} \times \sin(60)$ Equation I & II gives $F_{BC} = -7.77 \text{kN} \qquad F_{AB} = -3.77 \text{kN}$

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Summary

Sr. No.	Member	Force	Nature of force
1	AB	3.77kN	Compressive
2	AC	3.885kN	Tensile
3	ВС	7.77kN	Compressive



Calculate the force in each member of the loaded truss.









FBD of Joint 'A'

$$A \xrightarrow{F_{AE}} F_{AB}$$
$$R_{AY}$$

Equilibrium of Joint 'A' $\Sigma F_Y = 0 = R_{AY} + F_{AE} \sin(45)$ \rightarrow $F_{AE} = -1.414 \text{kN}$ $\Sigma F_X = 0 = F_{AE} \cos(45) + F_{AB}$ \rightarrow $F_{AB} = 1 \text{kN}$



FBD of Joint 'E'



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FBD of Joint 'B'





FBD of Joint 'C'





Equilibrium of Joint 'C'

$$\Sigma F_{Y} = 0 = F_{CD} + R_{CY} \longrightarrow F_{CD} = 1$$
kN



Summary

Sr. No.	Member	Force	Nature of force
1	AE	1.414 kN	Compressive
2	AB	1 kN	Tensile
3	BE	1 kN	Tensile
4	BC	2 kN	Tensile
5	BD	1.414 kN	Compressive
6	CD	1 kN	Tensile
7	DE	1 kN	Compressive


Section equilibrium method

- This method is also based upon the conditions of static equilibrium.
- This method is suitable when forces only in few of the members of the truss, particularly away from the supports are required.
- □ Conditions of equilibrium are $\sum F_x = 0$; $\sum F_y = 0$; $\sum M = 0$, Where $\sum M$ is the moments of all the forces about any point in the plane of the forces.
- Since we have only three equilibrium equation, only maximum three unknown forces can be found at a time. Therefore a care should be taken while taking a section that section line does not cut more than three members in which the forces are unknown.



Step by step procedure of section equilibrium method

- 1. Draw a FBD of an entire truss showing the external forces acting on it. (i.e. applied forces/loads and reactions)
- 2. Obtain the magnitude and direction of reactions by using the static equilibrium equations.
- 3. Draw an imaginary line which cuts the truss and passes through the members in which we need to find the forces. Care must be taken, to see that the section line must cut maximum of three members in which forces are unknown.



Step by step procedure of section equilibrium method

- 4. Prepare FBD of either part showing loads, reactions and internal forces in the members, which are cut by section line.
- 5. Apply the three conditions of equilibrium. If more than three unknowns are to be calculated, then two section lines can be selected and then the truss is to be solved by the above procedure.



Determine the forces in members *BC*, *BF*, *FJ*





Solution: Let's find support reactions



$$\begin{split} & \sum F_{X} = 0 = 50 - R_{AX} & \longrightarrow R_{AX} = 50 \text{kN} \\ & \sum M_{A} = 0 = R_{DY} \times 9 - 100 \text{ x } 3 - 100 \times 6 - 50 \times (3 \times \sqrt{3} \div 2) \rightarrow R_{DY} = 114.43 \text{kN} \\ & \sum F_{Y} = 0 = R_{AY} + R_{DY} - 100 - 100 & \longrightarrow R_{AY} = 85.57 \text{kN} \end{split}$$



Let's take appropriate section





Let's consider 'I' section



Equilibrium of section 'I'		
$\sum M_{\rm B} = 0 = -F_{FJ} \times (3 \times \sqrt{3} \div 2) - R_{AY} \times 3$	\rightarrow	<i>F_{FJ}</i> = -98.80kN
$\sum F_{\rm Y} = 0 = F_{BF} \times \sin(60) + R_{AY} - 100$	\rightarrow	<i>F_{BF}</i> = 16.66kN
$\Sigma F_{\rm X} = 0 = F_{BC} + F_{FJ} + F_{BF} \times \cos(60) - R_{AX}$	\rightarrow	<i>F_{BC}</i> = 140.47kN



Summary

Sr. No.	Member	Force	Nature of force
1	BC	140.47 kN	Tensile
2	BF	16.66 kN	Tensile
3	FJ	98.80 kN	Compressive



Friction

Friction forces are set up whenever a tangential force is applied to a body pressed normally against the surface of another



(a) Body A pressed against B







Friction force

- is the result of interaction of the surface layers of bodies in contact
- Two separate friction forces are given by

 $F_{s} = f_{s} N$ Where F_{s} is static friction force

 $F_{\kappa} = f_k N$ Where F_{κ} is kinetic friction force

□ $f_s \& f_k$ are static and kinetic coefficients of friction and are intrinsic properties of the interface between the materials in contact.



Main Properties of Friction Force (F)

- If there is no relative motion between A & B, then the frictional force (F) is exactly equal & opposite to the applied tangential force (T).
- This condition can be maintained for any magnitude of T between zero & certain limiting value F_s, called the *static friction force*.
- 3. IF T > F_s , sliding will occur.



Main Properties of Friction Force (F)

- If body A slides on body B, then F acting on body A will have a direction opposite to velocity of A relative to B, & its magnitude will be F_k, called the kinetic friction force.
- 5. F_s and F_k are proportional to normal force N.

$$F_S = f_S N$$
 ; $F_K = f_k N$



Both coefficients,

- Depends on materials of bodies in contact
- Depends on state of lubrication/contamination at the interface
- □ *Independent* of the area of the interface
- □ Independent of the roughness of the two interfaces
- □ *Independent* of time of contact and relative velocity respectively



Example

It is of interest to find the range of values of W which will hold the block of weight $W_o = 500$ N in equilibrium of the inclined plane if the coefficient of static friction is $f_s = 0.5$.



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Draw FBD of blocks A & B









Draw FBD of blocks A & B





Problem:

Frequently, a force F and a moment M act at the same point as shown in the figure for a coplanar system. Show that a coplanar force and moment may be replaced with an equal force displaced sideways a specified distance a = |M| / |F|. The are shown in the figure.





Problem:

A ladder of weight 390 N and 6 m long is placed against a vertical wall at an angle of 30^o as shown in figure. The coefficient of friction between the ladder and the wall is 0.25 and that between ladder and floor is 0.38. Find how high a man of weight 1170 N can ascend, before the ladder begins to slip.





Problem:

A circular cylinder A rests on top of two half-circular cylinders B and C, all having the same radius r. The weight of A is W and that of B and C is $\frac{1}{2}$ W each. Assume that the coefficient of friction between the flat surfaces of the half-cylinders and the horizontal tables top is f. determine the maximum distance d between the centers of the half-cylinders to maintain equilibrium.





Problem:

A freely pivoted light rod of length *I* is pressed against a rotating wheel by a force P applied to its middle. The friction coefficient between the rod and wheel materials if f. Compute, for both directions of rotation, the friction force F as a function of variables *I*, *P* and *f*, and any others which are relevant. One of these two situations is sometimes referred to as a friction lock. Which one and why?



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Problem:

Find the force carried in each bar of the hinged equilateral triangle when loaded as shown





Problem:

Find the force and moment which must be applied at *O* to hold the light bar shown in equilibrium.





Problem:

Estimate the force in link *AB* when the weight of the boat supported by the davit is 7 kN.





Problem:

Compare the forces F required to just start the 900-N lawn roller over a 75-mm step when

(a) the roller is pushed and

(b) (b) the roller is pulled





Problem:

A cantilever truss is loaded as shown in figure. Find the value of *W* which will produce a force of 150 kN magnitude in the member *AB*.





Problem:

By using the method of sections find the forces in the members BC, BF and BE of the cantilever truss shown in figure.





Problem:

Determine the forces in the six members of the truss shown





What we studied in Chapter-1

- Force
- Moment
- Types of loading
- Types of Support & related Reactions
- Free Body Diagrams
- Equilibrium Conditions
- Trusses
- Friction



Numerical are based on

- □ Free Body Diagrams
- □ Force/Reaction Determination
- Trusses
- Friction



References

- Introduction to Mechanics of Solids by S. H. Crandall et al (In SI units), McGraw-Hill
- 2. Mechanics of Materials, Beer and Johnston, McGraw-Hill